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Intermediate spin and quantum critical points, etc.

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Abstract

Unlike that of SO(3) or SU(2), the Lie algebra for SO(2), which defines intermediate spin, comprises only S_z and implies $S^{\pm}$ commute. In general, S_z has a continuous spectrum. This intermediate spin scheme can realized for the low energy excitations of a wide class of large spin magnets. A magnetic field provides the necessary time reversal symmetry breaking and controls the effective value of the spin $\tilde{S}$. Physical quantities are periodic in the equilibrium magnetization component induced by this field. In particular for one dimensional antiferromagnets there are periodic regions on the field axis for which the model is quantum critical while in two or three dimensions criticality is reduced to points.

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There is an elementary proof that the usual SO(3) and SU(2) spin algebra leads to values of the spin S which are either whole or half-integer. *Intermediate spin* arises in two dimensions and the context of anyons [1]. In the absence of time reversal invariance, the associated SO(2) algebra admits a continuous spectrum for S_z. For anyons the spectrum of S_z is determined by the statistical parameter α. In this Letter it will be shown that certain physically realizable large spin magnets with constituent whole or half-integer spin S, and with a suitably directed time-reversal-symmetry-breaking magnetic field, can approximate to the intermediate spin scheme with the magnitude of the magnetic field controlling the value of the effective statistical parameter α. The effective spin value $\tilde{S} = S - \alpha/2$ is determined by the physical value S in zero field and otherwise reflects the spectrum of $S_z = n - \alpha/2$; n an integer. Most thermodynamic properties are periodic in the field with a period H_p determined by the Hamiltonian parameters. Increasing the applied field by $H_p/2$ will convert a half-integer magnet into its whole integer equivalent. For arbitrary values of H the properties are, in general, intermediate between those of half and whole-integer spin. It will be shown explicitly for both ferro and antiferromagnetic systems with a large easy axis anisotropy energy D that most properties are strictly periodic with a period proportional to D. Antiferromagnets with a small or zero anisotropy energy also exhibit intermediate spin behavior for larger fields and sufficiently large spin values. Here the period H_p is proportional to the exchange energy J. The *exceptional* thermodynamic quantity, in both cases, is the uniform magnetization M_0. This is a steadily increasing function of H (or α) with *plateaus* whenever there is a gaped "whole integer" phase. For a uniform system such plateaus are separated by increments in M_0 which correspond to a single μ_B per physical spin.

Intermediate spin systems can have various dimensions. Zero dimensional models corresponding to the tunneling of the magnetization of nano-particles. For *zero field* Loss et al [2] and von Delft and Henley [3] have shown the ground doublet tunnel splitting is strictly zero for half-integer spin when, for the same model and for whole-integer spin, this splitting is finite. Hard axis nano-magnets are explicitly covered by these results and with a finite field parallel to the hard axis must also manifest intermediate spin properties. *Both* on

the basis of the general intermediate spin considerations*and* from the direct solution of the problem, it is here shown that this tunnel splitting is indeed periodic in the field. Tunneling in antiferromagnets has also been studied [7,8]. It has been demonstrated explicitly that the anticipated periodic behavior is exhibited even for systems with a small or zero explicit anisotropy energy.

For one dimension and *zero field*, Haldane [4] has conjectured, for antiferromagnetic chains for which the whole integer spin version exhibits a gap, the half-integer equivalent does not. Hard axis such spin chains should conform with this conjecture but in a field again must also reflect intermediate spin properties. Using well established results, the large anisotropy limit can be solved. On this basis it can again be *explicitly* demonstrated that a certain field $H_p/2$ converts a whole into half-integer spin problem. Whatever the whole or half-integer nature of the physical spins, these models have gapless "half-integer" quantum critical regions surrounding periodic points on the field axis of their phase diagram. The rest of the field axis comprises gaped "whole integer phases". Results for antiferromagnets chains with a small or zero anisotropy energy are more difficult to demonstrate. However the work of Oshikawa [5] and Totsuka [6] on smallish half-integer spin chains shows that "whole integer" plateaus exist thereby supporting the general idea to be present here that a magnetic field leads to spin transmutation.

In two or three dimensions, where the Haldane conjecture does not apply, there are similar periodic phases diagrams with, e.g., half-integer phases which terminate with field determined quantum critical points. Given the current high level of interest in quantum critical points the ready availability of high fields this suggests any number of exciting experimental possibilities.

That intermediate spin properties can arise in physical whole or half-integer spin systems has implications for certain concepts based on topological considerations. Both the results of Loss et al [2] and von Delft and Henley [3] and the Haldane conjecture [4] are explained [9] in terms of the so called *topological term* in the effective action. In the usual notation this is $iS(1-\cos\theta)\dot{\phi}$ and the differences between whole and half-integer spin flow from the value of

the spin S in this expression. These results and conjectures correspond to Hamiltonians with time reversal symmetry while the realization of intermediate spin implies this symmetry is broken. It must be that the same interference effect which destroy the gaps for half-integer spin *can and do* occur periodically as a function of applied field even for systems constituted of physical whole-integer spins.

The necessary dimensional reduction SO(3) or SU(2) $\Rightarrow$ SO(2) is most easily accomplished by a hard axis anisotropy which forces spins to lie in the $x-y$-plane. However for *antiferromagnets* the field alone suffices. As is well known, for ordered phases, a magnetic field H_{sf} causes a "spin-flop" transition. What is perhaps not appreciated is that this can be considered as a competition between the true anisotropy energy and an effective hard axis anisotropy energy $D_H \sim H^2/J$, J the exchange, induced by the external field H. With $\vec{H}$ along the easy axis the spin-flop occurs when $D_H \sim D$ a criterion which correctly leads to $H_{sf} \sim \sqrt{DJ}$. For the intermediate spin scheme to apply it must be that this induced hard axis anisotropy D_H is larger than the exchange, i.e., that $H > J$. When this inequality is well obeyed, and for large enough spin, most thermodynamic properties are again periodic with plateaus in the magnetization.

It is again emphasized that the models which approximate to the intermediate spin scheme are constituted of whole and/or half-integer spins S and that the point of the exercise is no more to disprove the basic theorem on spin values than the demonstration that condensed matter systems can exhibit the properties of anyons is intended to disprove that these systems are ultimately composed of bosons and fermions. The Lie algebra of SO(2) has only the single operator S_z which is the generator of rotations about the z-axis. Consider a rotational degree of freedom ϕ of two anyons [1]. In a non-singular gauge define: $S_z = \frac{1}{2}\left(-i(d/d\phi) - \alpha\right)$, where α is the statistical parameter and where the eigenfunctions are $F_\ell = e^{2is\phi}$, $s = 0, \pm 1, \pm 2 \ldots$ to give a spectrum $S_z = s - \alpha/2$. The raising and lowering operators are simply $S^{\pm} = \tilde{S} e^{2i\theta}$, where $\tilde{S}$ is spin value to be defined below. The spin algebra is then uniquely:

$$[S_z, S^{\pm}] = \pm S^{\pm} \quad \text{and} \quad [S^+, S^-] = 0 \tag{1}$$

These commutation rules, which are equivalent to $[\phi, p_\phi] = i\hbar$ and $[\phi, \phi] = 0$, are here taken as the *definition* of intermediate spin. Time reversal symmetry, $(S_z \to -S_z)$ implies that the spectrum of S_z is whole or half integer, however in general this is *not* the case, i.e., this spectrum is continuous. An intermediate spin model comprises a Hamiltonian $\mathcal{H}$ given as a function of S_z and $S^{\pm}$. It is then observed that if $-\alpha/2$ is the smallest absolute value in the applicable spectrum of S_z, this Hamiltonian will only couple eigenstates with $S_z = n - \alpha/2$ where n is an integer. Finally, the spin value $\tilde{S}$ is *chosen* to be $S - \alpha$, where S is an integer. (A different choice might be absorbed into a re-definition of the interaction parameters in the transverse part of $\mathcal{H}$¿) This $\tilde{S}$ is an eigenvalue of S_z and characterizes a solution of $\mathcal{H}$. With physical spin S models $\tilde{S} = S$ when $\alpha = 0$.

Given that Eqns. (1) define intermediate spin, it is necessary to show that this represents an interesting possibility in two ways. First that there real physical systems composed out of whole and/or half-integer spins for which the intermediate spin description is valid as a good approximation, and second that such systems have the non-trivial properties described above.

That this is the case is easiest to demonstrate for the above mentioned class of *hard axis* magnets with not *too* small values of the constituent whole or half-integer spin S [10]. The role of the large anisotropy energy is to enforce the dimensional reduction discussed above. The hard direction is taken to be the z-axis and the class of Hamiltonians is defined by:

$$\mathcal{H} = g\mu_B H \sum_n S_{z,n} + D \sum_n {S_{z,n}}^2 + \mathcal{H}_1(\vec{S}_n) \tag{2}$$

where D is a *positive* anisotropy energy which is sufficiently large that

$$S^2 D \gg \; g\mu_B H, \tag{3}$$

and

$$S^2 D \gg < \mathcal{H}_1 >, \tag{4}$$

and where the interaction Hamiltonian $\mathcal{H}_1$ is a fairly general scalar function of the $\vec{S}_n$, see below. The expectation value is for the low lying states of interest and the indice n labels the constituent spins each of magnitude S. It is trivial that the magnetic field can be absorbed into the anisotropy term via a redefinition of the spin operator

$$\hat{S}_{n,z} \Rightarrow \hat{S}_{n,z} - \frac{\alpha}{2}; \qquad \alpha = \frac{g\mu_B H}{D} \tag{5}$$

and an unimportant shift in energy. The spins are forced to lie near the $x-y$-plane and, provided the *physical* spin value is sufficiently large, i.e., if

$$S \gg \alpha, \tag{6}$$

for low energy states, the matrix elements of $S_n^{\pm}$ can be replaced by S, which is equivalent to the commutation rule $[S_n^+, S_{n'}^-] = 0$ of Eqn. (1). Clearly the substitution $\hat{S}_{n,z} \Rightarrow \hat{S}_{n,z} - \alpha/2$ has no effect on $[S_n^+, S_{n'}^-] = 0$ and leaves unchanged $[\hat{S}_{n,z}, S_n^{\pm}] = \pm S_n^{\pm}$. If the original basis is re-labeled $|S_{n,z} > \Rightarrow |S_{n,z} - \alpha/2 >$ then $\hat{S}_{n,z}|S_{n,z} - \alpha/2 >= (S_{n,z} - \alpha/2)|S_{n,z} - \alpha/2 >$ and the whole algebra is that for intermediate spin with an effective spin

$$\tilde{S} = S - \alpha/2. \tag{7}$$

There are necessarily some restrictions on the interaction term $\mathcal{H}_1$. Uniform quadratic terms such as $\sum_{n,n'} J^{\|}_{n,n'} S_{n,z} S_{n',z} \Rightarrow \sum_{n,n'} J^{\|}_{n,n'} S_{n,z} S_{n',z} + h_1 \sum_n S_{n,z} + \text{const.}$, as $\hat{S}_{n,z} \Rightarrow \hat{S}_{n,z} - \alpha/2$ and cause no problems since the $h_1 = zJ^{\|}/2 \ll g\mu_B H$, z the number of neighbors, results only in a small modification to the definition of the effective applied field. The corresponding transverse terms do not change, i.e., $(1/2)\sum_{n,n'} J^{\perp}_{n,n'}(S_n^+ S_{n'}^- + H.c.)$ transforms to itself. Similarly for an isotropic expression $\sum_{n,n'} J_{n,n'}(\vec{S}_n \cdot \vec{S}_{n'})^n$ there are unimportant small modifications to both the field and the anisotropy energy. However, e.g., $S_{n,z}(S_{n'}^+ + S_{n'}^-)$, which reflects a lowering of symmetry, generates an apparent transverse applied field and implies that a small component of the applied field must be used to cancel such terms. However not all terms which lower the symmetry are problematic. In particular the expression $K_\perp \sum_n S_{n,x}^2$, which is important for tunneling problems transforms into itself. Thus, while

perhaps not a completely general function of the spin operators, the allowed operators $\mathcal{H}_1$ cover a wide variety of interesting problems.

Turning to the more general case when the anisotropy energy is small or zero, it is observed that $\hat{S}_{n,z} \Rightarrow \hat{S}_{n,z} - \alpha/2$ can apparently still be used to eliminate the magnetic field terms since as effectively noted above $J\sum \vec{S}_n \cdot \vec{S}_{n'} \Rightarrow J\sum \vec{S}_n \cdot \vec{S}_{n'} + h_1 \sum_n S_{n,z} +$ const. *Unfortunately* in the absence of the large anisotropy term there is no assurance that the matrix elements of $S^{\pm}$ reduce to S as was the case above. An appropriate, zero dimensional, tunneling problem has been studied in some detail [7,8] and the same principles can be applied to arbitrary dimensions. An abbreviated fashion to obtain the their results goes as follows: Effectively the model comprises two spin interacting via a $J\vec{S}_1 \cdot \vec{S}_2$ so the ground state energy is $F(m) = (J/2)[m(m+1) - 2S(S+1)] - m(g\mu_B H)]$ where m is the z-quantum number for the total magnetisation. The aim of the intermediate spin transformation $\hat{S}_{n,z} \Rightarrow \hat{S}_{n,z} - \alpha/2$ is to eliminate the time-reversal-breaking terms and as a result the equilibrium magnetization $g\mu_B m_0$ is automatically $g\mu_B \alpha/2$, i.e., the statistical parameter is determined directly by the equilibrium magnetization. For the tunneling problem the equilibrium magnetization gives $\alpha/2 = (g\mu_B H/J) - (1/2)$, and a ground state energy which contains a $\sim -[(g\mu_B H)^2/2J]$ field generated anisotropy energy. That there is a $\pi/2$ phase shift corresponding to the 1/2 is confirmed by both direct calculation and numerical solution [7,8]. These results also show that the true intermediate spin regime, with this phase shift, only sets in for larger fields such that $g\mu_B H$ is a few times J illustrating the necessary development of the field induced hard axis anisotropy. The result $H_p = (J/g\mu_B)$ also agrees with direct calculation. Rather generally, antiferrromagnet systems in a finite field have a larger transverse that longitudinal susceptibility, develop a field induced hard axis anisotropy energy, and undergo spin-flop transitions. When this energy is large compared with the exchange energy the system will exhibit intermediate spin properties with in general

$$\alpha = 2m_0 = 2\frac{\chi H}{g\mu_B} \tag{8}$$

where χ is the magnetic susceptibility. While in general, as a function of H, e.g., whole

integer plateaus will be phase shifted as compared with the $H = 0$ point, such plateaus and half-integer points will always be exactly displaced by units of $\mu_B/2$ along the m_0 axis. For low fields some plateaus might be missing because a large enough anisotropy energy has not yet been developed and the systems is not in the intermediate spin regime.

The basic result of this Letter is the observation that, with some qualifications on the interaction and when the intermediate spin inequalities are well satisfied, many large spin magnets have physical properties which are periodic in the equilibrium magnetization. The effective value of the spin $\tilde{S} = S - \alpha/2$ where the statistical parameter α is determined by the equilibrium magnetization via the applied field. Increasing the equilibrium magnetization by μ_B per spin maps the problem back to itself while an increase by $\mu_B/2$, e.g., converts a half-integer problem into its whole integer equivalent

As an example, consider first the zero dimensional problem of a small easy plane ferromagnetic nano-particle. This is modeled by a single large spin subject to the external and anisotropy fields [11], i.e., the Hamiltonian is $\mathcal{H} = g\mu_B H S_z + K_{\|} {S_z}^2 + K_{\perp} {S_x}^2$ where without loss of generality it is assumed that $|K_{\|}| > |K_{\perp}|$. (Here $K_{\|} \equiv D$.) For an easy plane magnet $K_{\|} > 0$. This is of the class of Eqn. (2) with a single physical spin S and $\mathcal{H}_1 = K_{\perp} {S_x}^2$.

The problem is formulated [12] in terms of (Abrikosov) auxiliary particles. A basis $|S_z> \equiv |n>$ is chosen. Then an auxiliary particle, a fermion f_n, is associated with each state via the mapping $|n> \rightarrow f_n^\dagger| >$ where $| >$ is a non-physical vacuum without any auxiliary particles. Defined is a bi-quadratic version of an operator $\hat{O}$ via: $\hat{O} \rightarrow \sum_{n,n'} f_n^\dagger < n|\hat{O}|n' > f_{n'}$. The constraint $Q = \sum_n \hat{n}_n = \sum_n f_n^\dagger f_n = 1$. It has been shown [12] that such schemes preserve all operator multiplication rules including commutation rules. The replacement rule is applied to the Hamiltonian $\mathcal{H}$ to yield, setting $h = g\mu_B H$:

$$\begin{gathered}\mathcal{H} = \textstyle\sum_n \Big(K_{\|} n^2 - nh + \\ \tfrac{1}{4} K_{\perp} [(M_n^{n+1})^2 + (M_n^{n-1})^2] \Big) f_n^\dagger f_n \\ + \tfrac{1}{4} K_{\perp} \textstyle\sum_n M_n^{n+1} M_{n+1}^{n+2} (f_{n+2}^\dagger f_n + H.c.),\end{gathered} \tag{9}$$

where the $M_n^{n+1} = [S(S+1) - n(n+1)]^{1/2}$ are the matrix elements of $S^{\pm}$. This is *two*

disconnected tight binding chains of spinless fermions $f_n^\dagger$. The constraint $Q = 1$ implies this is a single particle problem. The "site" indices are whole or half-integers following the nature of the constituent spins.

The intermediate spin regime requires inequality (4) which translates to $K_\parallel > K_\perp$. However usually considered by others [2,3,13] is the "tunneling regime" which implies $K_\parallel < S^2 K_\perp$ so large values of S are implied. The wave-function is confined near $n = 0$ and the problem is, to an excellent approximation, that of a single particle in a harmonic well. The spin value is reflected only by the matrix elements $M_n^{n+1} \approx S$, again to a very good approximation. It is interesting that there is "no under the barrier" tunneling in this representation of the problem. The small "tunneling" energy difference displayed below is actually the difference in energy between the ground states on the two chains. It arises because the harmonic problem on a discrete lattice depends at an exponential level on the disposition of the sites relative to the origin of the harmonic potential. In general this disposition is different for these two chains.

It is easily appreciated that (i) the only difference between the whole and half-integer problems is a shift in the "sites" by 1/2 and (ii) an exactly similar shift $h/2K_\parallel$ is induced (with and unimportant shift in the total energy) by an applied field. It follows that the applied field causes the effective spin value to be continuous and physical properties to be periodic as described above. This problem has been solved directly using the above described auxiliary particle formulation [7]. The *result* for the tunnel splitting is:

$$\begin{aligned} &\delta E = 4|\cos(\pi\tilde{S})|\sqrt{\frac{2}{\pi}}\omega_0 S_f^{1/2} e^{-S_f}; \\ &S_f = 2S\sqrt{(K_\perp/K_\parallel)}; \quad \omega_0 = S\sqrt{K_\perp/K_\parallel}, \end{aligned} \tag{10}$$

where $\tilde{S} = S - \alpha/2$. This is *identical* to the zero field results [2,3] *except* for the anticipated change $S \to \tilde{S}$. (Garg [13] had previously shown the tunnel splitting is quasi-periodic in the field. However his development and interpretation of this fact was quite different and he did not give an analytic result for δE which might be compared with the zero field result.)

As already mentioned, the tunneling problem for an antiferromagnet with a *small or zero*

anisotropy energy has also been solved and exhibits the expected oscillations with period $H_p = J/g\mu_B$ [7,8], in the present notation.

For one dimensional chains and a large easy axis anisotropy, a simple case corresponds to the general Hamiltonian Eqn. (2) with $\mathcal{H}_1 = J\sum_n \vec{S}_n\cdot\vec{S}_{n+1}$; $J > 0$, i.e., an antiferromagnetic Heisenberg model *but* with a large anisotropy energy. Here the inequality (4) is $D > J$. Unfortunately there is no available solution unless $D \gg S^2 J$, corresponding to a limiting or fixed point. In this limit well established solutions can can be used to infer the relevant phase diagram both for the vicinity of the fixed point and less directly for the more general case [15] when $K_\| > J$. A similar fixed point approach has been shown useful for the tunneling problem [7,14].

It is necessary to add a *real space site* index n to the Fermi operators: $f_{n,m}$. In the limiting case, $D \gg S^2 J$, zero field, and *whole integer spin* the ground state of the *dominant* anisotropy term, in zero field for each real space site, is $|S_{nz} = 0 >= f_0^{\dagger n}| >$. *The interaction is a perturbation* and *in the limit* the ground state is simply $\prod_n f_0^{\dagger n}| >$, reflecting a singlet at each site. There is a, trivial and large, "Haldane gap" in the excitation spectrum of magnitude $\sim K_\|$ [15].

In contrast *for half-integer spin* there is a doublet ground state of $K_\| {S_{nz}}^2$ comprising $|S_{nz} = \pm 1/2 >= f_{\pm 1/2}^{n\dagger}| >$ for each site and for $K_\| \gg S^2 J$ and small fields, $\alpha < 1$, the effective Hamiltonian for these doublets is

$$\mathcal{H}_a = g\mu_B H \sum_n s_{n,z}$$
$$+J\sum_n \left[s_{n,z}s_{n+1,z} + S(s_n^+ s_{n+1}^- + H.c.)\right] \tag{11}$$

where $s_{n,z} = (1/2)[f_{n,1/2}^\dagger f_{n,1/2} - f_{n,-1/2}^\dagger f_{n,-1/2}]$ and $s_n^+ = f_{n,1/2}^\dagger f_{n,-1/2}$. These operators correspond to a regular SU(2) spin algebra with $S = 1/2$ and $\mathcal{H}_a$ is the integrable anisotropic Heisenberg model *without* an explicit anisotropy energy *but* with $J_\perp/J_\| = S$. For small fields H this model is gapless and *critical* [18]. All correlation functions decay as simple power laws [18], i.e.,

$$< s_{n+1,z}s_{n,z} > - < s_z >^2 \sim \frac{1}{n^\theta};$$

$$< s_{n+1}^{+} s_n^{-} > \sim \frac{1}{n^{1/\theta}} \tag{12}$$

where for this small field regime $\theta = (\pi/2\eta)(1 + \alpha_1 H^2)$; $\alpha_1 = (1 - (2\eta/\pi))^2[\cot(\pi\eta/(\pi - 2\eta))/8\eta \sin^2 2\eta]$ and where the anisotropy parameter η is defined by $\cos 2\eta = J_{\parallel}/J_{\perp} = (1/S)$ and is of the order of, but slightly greater, than $\pi/4$.

There is [18] a *critical field* defined by $H_c = J \sin^2 \eta$ and for fields less that H_c the system remains critical. Approaching H_c, $\theta = 2 + 4(\pi \tan \eta \tan 2\eta)^{-1}\sqrt{H_c - H}$ while for $H > H_c$ the magnet is ordered "ferromagnetically".

Turning to larger finite fields, the model can be equally well be solved in the vicinity of fields which are multiples of $(H_p/2) = K_{\parallel}/g\mu_B$. Specifically for half-integer spin when $H = K_{\parallel}/g\mu_B$, at each site the state $f_{n,1/2}^{\dagger}| >$ is the ground state of the anisotropy energy with $f_{n,-1/2}^{\dagger}| >$ and $f_{n,3/2}^{\dagger}| >$ degenerate, at an energy $K_{\parallel}$ higher. For low energies the problem maps exactly onto the whole integer equivalent with again the large gap $\sim K_{\parallel}$. There is, of course, a ferromagnet moment of $\sim g\mu_B/2$ per site.

Doubling the field so $H = 2K_{\parallel}/g\mu_B$, causes the states $f_{n,1/2}^{\dagger}| >$ and $f_{n,3/2}^{\dagger}| >$ to become degenerate so that again to a good approximation the Hamiltonian (11) describes the situation except $H \to H - 2K_{\parallel}/g\mu_B$ and $s_{n,z} = (1/2)[f_{n,3/2}^{\dagger} f_{n,3/2} - f_{n,1/2}^{\dagger} f_{n,1/2}]$ with $s_n^{+} = f_{n,3/2}^{\dagger} f_{n,1/2}$. The model is critical in the interval $2K_{\parallel}/g\mu_B - H_c$ to $2K_{\parallel}/g\mu_B + H_c$ and the only significant difference with zero field is the existence of an average ferromagnetic moment of $\sim g\mu_B$ per site.

With each level crossing at $H = 2nK_{\parallel}/g\mu_B$ the model maps back to that in zero field except for a moment of $ng\mu_B$ per site, while for fields of $H = (2n+1)K_{\parallel}/g\mu_B$ the map is to whole integer spin with an additional moment of $\sim (2n+1)g\mu_B/2$. There is a "half-integer" critical phase delimited by the points $2nK_{\parallel}/g\mu_B \mp H_c$. This is illustrated in Fig. 1. The phase diagram for constituent whole integer spin is similar with a shift of $K_{\parallel}/g\mu_B$ along the field axis and with background moments $\sim (2n + 1)g\mu_B/2$ around the effective half-integer points.

The solutions near the half-integer points connect smoothly with the singlet ground states

valid near the center of the whole-integer phases. For example, still for a half-integer chain, for fields modestly larger than H_c the "ferromagnetic" ground state from the exact solution is, in the current language, approximately $\prod_n f^\dagger_{n,1/2}| >$ which is just the singlet ground state for the next whole-integer phase, providing a connection between the two solutions. It is less obvious that Fig. 1 also describes the phase diagram when only the weaker inequality $K_\parallel > J$ is satisfied. The periodic behavior originates from level crossing at fields which are a multiples of H_p. The inequality guarantees that the ground and low lying states ψ_α are admixtures of states $|n, m >$ with small coefficients for states with $|m| \sim S$ because these cost too much energy. The change in these ψ_α when $H \rightarrow H + H_p$ can be accounted for to a good approximation by the change $|n, m > \rightarrow |n, m-1 >$. This way of re-stating that a finite field is equivalent to a translation in the origin of the potential strongly suggests that the weaker inequality is sufficient for the models to exhibit periodic behavior, providing one stays within the "large-D" phase [15].

In two or three dimensions there are rather too many possibilities to be enumerated here, however the basic structure along the H axis of the phase diagram must always be very similar. If, e.g., the system is composed of half-integer spins then further "half-integer phases" can occur whenever the magnetic field increases the uniform magntization by μ_B per spin. Well within the intermediate spin regime these points will be periodic in the field. Midway between these half-integer phases are "whole-integer phases". There remains only critical *points* at the limits of the "half-integer phases".

The equivalence between the zero field $S = 1/2$ half-integer problem and a whole integer spin model with an large anisotropy energy and a suitable magnet field is not new. For spin $S = 1$ this possibility was analyzed at the molecular field level long ago [16,17]. Refs. [5,6] on the other hand show some small spin half-integer spin systems exhibit magnetization plateaus. Both sets of results can be interpreted as supporting the idea of the transmutation of spin described here.

Experimental examples of the predicated periodic behavior are perhaps difficult to find because of the required large fields and spin values. Current laboratory fields of $H_{\max} \sim 30$T

implies low temperature magnets. Small values of the exchange are not particularly sought after but certainly exist in the region 100mK to 10K making the search for intermediate spin behavior possible with current facilities. The evident experimental signature of intermediate spin is the periodic behavior of nearly all thermodynamic quantities and in particular the field derivative of the magnetization and the specific heat. The only example of such a quasi-periodic behavior of which the author is aware is that of Taft et al [19] for Fe_{10} which is a zero dimensional example with small anisotropy. For Mn_{12}, Thomas et al., and Friedman et al, [20] observe periodic behavior but for quite different reasons. Magnetization plateaus which might be interpreted as whole $\Leftrightarrow$ half-integer mutations have been observed for small spin systems: see [5,6,16,17].

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fact it is known that this “large-D” phase is separated from the zero field “Haldane” phase by a critical value $D_c = 2J$, see e.g., ref. 5 for discussion and references. However, as shown in Refs. 2 and 3, precisely for the case of a single spin, the presence of such a gap for an integer single spin and its absence for half-integer spin is of the same topological origin as is the Haldane conjecture, i.e., the present fixed point for zero field *does* have a Haldane gap albeit of a rather trivial origin.

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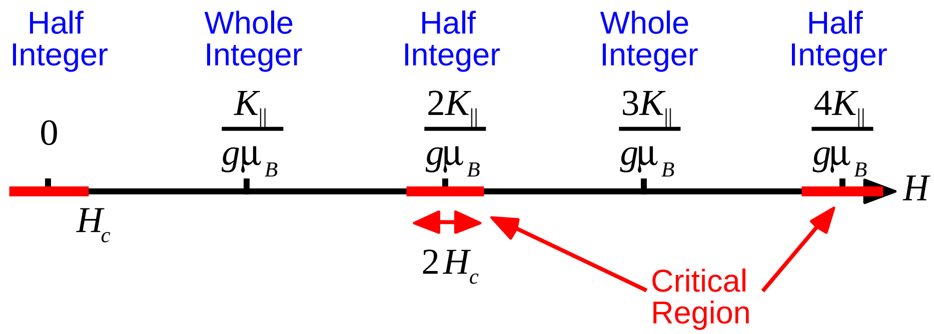
FIGURES

FIG. 1. The zero temperature field axis of the phase diagram for the half-integer constituent spins. The "half-integer" phases are centered at $H = 0$, $(2D/g\mu_B)$, $(4D/g\mu_B)\ldots$ while the "whole-integer" phases have their centers at $H = (D/g\mu_B)$, $(3D/g\mu_B)\ldots$. The former phases have the relatively small width $2H_c$ while latter occupy the rest of the phase diagram. The magnetic moments at the center of the phases are 0, $g\mu_B\, 2g\mu_B \ldots$ and $g\mu_B/2\; 3g\mu_B/2\ldots$ respectively. However the field derivative $\partial M/\partial H$ and most other thermodynamic quantities are periodic along this axis.



BARNES Figure 1